

# Unifying phantom inflation with late-time acceleration: scalar phantom-non-phantom transition model and generalized holographic dark energy

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The unifying approach to early-time and late-time universe based on phantom cosmology is proposed. We consider gravity-scalar system which contains usual potential and scalar coupling function in front of kinetic term. As a result, the possibility of phantom-non-phantom transition appears in such a way that universe could have effectively phantom equation of state at early time as well as at late time. In fact, the oscillating universe may have several phantom and non-phantom phases. The scalar factor in the kinetic term does not play any role in each of two phase and can be absorbed into the redefinition of the scalar field. Right on the transition point, however, the factor cannot be absorbed into the redefinition and play the role to connect two phases smoothly. As a second model we suggest generalized holographic dark energy where infrared cutoff is identified with combination of FRW parameters: Hubble constant, particle and future horizons, cosmological constant and universe life-time (if finite). Depending on the specific choice of the model the number of interesting effects occur: the possibility to solve the coincidence problem, crossing of phantom divide and unification of early-time inflationary and late-time accelerating phantom universe. The bound for holographic entropy which decreases in phantom era is also discussed.

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## I. INTRODUCTION

The recent astrophysical data indicate that effective equation of state parameter  $w_{\text{eff}}$  of dark energy lies in the interval:  $-1.48 < w_{\text{eff}} < -0.72$  [1]. In other words, it is quite possible that current universe lives (or enters) at effective phantom era (for review of observational data indicating to phantom late universe see [2] and last ref. from [1] and for recent discussion of various approaches to late-time phantom cosmology, see [3, 4] and references therein). However, it is not clear how to relate the late-time phantom cosmology with early-time inflation. For instance, the transition from decelerating phase to dark energy universe is not yet well understood (possibly because it is not clear what is dark energy itself). Nevertheless, there are attempts to unify the early time inflation where phantoms are essential with accelerated phantom universe [5]. The unified inflation/acceleration universe occurs for some models of modified gravity [6] as well as for complicated, non-standard equation of state (EOS) for the universe [7] (for recent discussion of such (phantomic) EOS, see [8, 9]). The attempts to use phantoms in early universe may be found also in [10].

In the present work we suggest the scenario where within the same theory, quite naturally there occurs both phenomena: early-time phantom inflation and late-time phantom acceleration. The circles of phantom-dominated and non-phantom dominated epoch in such universe suggest that probably the universe is (partially) oscillating. In the next section we consider gravity-scalar theory with scalar-dependent coupling

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in front of kinetic term and scalar potential. We should note that the scalar factor in the kinetic term does not play any role each in the phantom or non-phantom phase and can be absorbed into the redefinition of the scalar field. Right on the transition point, however, the factor cannot be absorbed into the redefinition and play the role to connect two phases smoothly. In the number of explicit examples it is demonstrated how transitions between phantom and non-phantom phases occur and that two phases are smoothly connected with each other. As a result, there occurs the universe which contains at least two phantom phases corresponding to early time inflation and late time acceleration. The bridge between phantom phases correspond to the standard non-phantom cosmology (radiation/matter dominated, expanding or shrinking one). In oscillating universe there may emerge multiple phantom/non-phantom transitions (eras).

Section three is devoted to the study of generalized holographic dark energy where infrared cutoff depends on the combination of Hubble rate, the particle and future event horizons, life-time of the universe and even cosmological constant. Here, the analogy with AdS/CFT correspondence may be pointed out: there also IR or UV cutoffs represent some combination (depending on the order of the expansion in large  $N$ ). It is shown that in such model quite naturally the crossing of phantom divide occurs. When including dark matter, the natural solution of coincidence problem follows. Finally, it is demonstrated that the unification of phantom inflation with phantom dark energy universe is also possible. Some summary and outlook are given in the last section.

## II. PHANTOM INFLATION AND LATE-TIME ACCELERATION IN SCALAR THEORY

In the present section we will discuss usual gravity with scalar field. The possibility to have unified phantom inflation with phantom late-time acceleration is shown. This occurs via phantom-non-phantom transition. Let us start from the following action:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \omega(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) \right\} . \quad (1)$$

Here  $\omega(\phi)$  and  $V(\phi)$  are functions of the scalar field  $\phi$ . Such scalar theory reminds about self-coupled dilaton [12] or about sigma-model. The spatially-flat FRW metric is

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^3 (dx^i)^2 . \quad (2)$$

The scalar field  $\phi$  only depends on the time coordinate  $t$ . Then the FRW equations are given by

$$\frac{3}{\kappa^2} H^2 = \rho , \quad -\frac{2}{\kappa^2} \dot{H} = p + \rho . \quad (3)$$

Here the energy density  $\rho$  and the pressure  $p$  are

$$\rho = \frac{1}{2} \omega(\phi) \dot{\phi}^2 + V(\phi) , \quad p = \frac{1}{2} \omega(\phi) \dot{\phi}^2 - V(\phi) . \quad (4)$$

By combining (3) and (4), one obtains

$$\omega(\phi) \dot{\phi}^2 = -\frac{2}{\kappa^2} \dot{H} , \quad V(\phi) = \frac{1}{\kappa^2} (3H^2 + \dot{H}) . \quad (5)$$

The interesting case is that  $\omega(\phi)$  and  $V(\phi)$  are defined in terms of single function  $f(\phi)$  as

$$\omega(\phi) = -\frac{2}{\kappa^2} f'(\phi) , \quad V(\phi) = \frac{1}{\kappa^2} (3f(\phi)^2 + f'(\phi)) . \quad (6)$$

Hence, the following solution may be presented

$$\phi = t , \quad H = f(t) . \quad (7)$$

One can check the solution (7) satisfies the scalar field equation:

$$0 = \omega(\phi)\ddot{\phi} + \frac{1}{2}\omega'(\phi)\dot{\phi}^2 + 3H\omega(\phi)\dot{\phi} + V'(\phi) . \quad (8)$$

Then *any* cosmology defined by  $H = f(t)$  in (7) can be realized by (6).

As clear from the first equation (5), when  $\dot{H}$  is positive, which corresponds to the phantom phase,  $\omega$  should be negative, that is, the kinetic term of the scalar field has non-canonical sign. On the other hand, when  $\dot{H}$  is negative, corresponding to the non-phantom phase,  $\omega$  should be positive and the sign of the kinetic term of the scalar field is canonical. If we restrict in one of phantom or non-phantom phase, the function  $\omega(\phi)$  can be absorbed into the field redefinition given by

$$\varphi = \int^\phi d\phi \sqrt{\omega(\phi)} , \quad (9)$$

in non-phantom phase or

$$\varphi = \int^\phi d\phi \sqrt{-\omega(\phi)} , \quad (10)$$

in phantom phase. Usually, at least locally, Eq.(9) or Eq.(10) can be solved with respect to  $\phi$  as  $\phi = \phi(\varphi)$ . Then the action (1) can be rewritten as

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R \mp \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \tilde{V}(\varphi) \right\} . \quad (11)$$

Here

$$\tilde{V}(\varphi) \equiv V(\phi(\varphi)) . \quad (12)$$

In the sign  $\mp$  of (11), the minus sign corresponds to the non-phantom phase and the plus one to the phantom phase. Then both of  $\omega(\phi)$  and  $V(\phi)$  in the action (1) do not correspond to physical degrees of freedom but only one combination given by  $\tilde{V}(\varphi)$  has real freedom in each of the phantom or non-phantom phase and defines the real dynamics of the system. The redefinition (9) or (10), however, has a discontinuity between two phases. When explicitly keeping  $\omega(\phi)$ , the two phases are smoothly connected with each other (kind of phase transitions). Hence, the function  $\omega(\phi)$  gives only redundant degree of freedom and does not correspond to the extra degree of freedom of the system (in the phantom or non-phantom phase). It plays the important role just in the point of the transition between the phantom phase and non-phantom phase. By using the redundancy of  $\omega(\phi)$ , in any physically equivalent model, one may choose, just for example,  $\omega(\phi)$  as  $\omega(\phi) = \omega_0 (\phi - \phi_0)$  with constants  $\omega_0$  and  $\phi_0$ . If we further choose  $\omega_0$  to be positive, the region given by  $\phi > \phi_0$  corresponds to the non-phantom phase, the region  $\phi < \phi_0$  to the phantom phase, and the point  $\phi = \phi_0$  to the point of the transition between two phases.

Since the second FRW equation is given by

$$p = -\frac{1}{\kappa^2} (2\dot{H} + 3H^2) , \quad (13)$$

by combining the first FRW equation, the effective equation of state parameter  $w_{\text{eff}}$  looks as

$$w_{\text{eff}} = \frac{p}{\rho} = -1 - \frac{2\dot{H}}{3H^2} . \quad (14)$$

After this discussion, one may consider some toy models to realize above phantom-non-phantom transition to unify the phantom inflation with late-time phantom acceleration.

As a first example, we consider the following model

$$f(\phi) = \frac{\alpha}{3}(T_0 + \phi)^3 - \beta(T_0 + \phi) + \gamma , \quad \gamma \equiv -\frac{\alpha}{3}T_0^3 + \beta T_0 , \quad (15)$$

with the constants  $\alpha$ ,  $\beta$ , and  $T_0$ , which give

$$\begin{aligned}\omega(\phi) &= -\frac{2}{\kappa^2} \{ \alpha(T_0 + \phi)^2 - \beta \} , \\ V(\phi) &= \frac{3}{\kappa^2} \left\{ \frac{\alpha^2}{3} (T_0 + \phi)^6 - 2\alpha\beta(T_0 + \phi)^4 + \alpha\gamma(T_0 + \phi)^3 \right. \\ &\quad \left. + (\alpha + 3\beta^2) (T_0 + \phi)^2 - 2\beta\gamma(T_0 + \phi) + 3\gamma^2 \right\} .\end{aligned}\tag{16}$$

Then the solution can be given by

$$H = \frac{\alpha}{3} (T_0 + t)^3 - \beta(T_0 + t) + \gamma , \quad \phi = t ,\tag{17}$$

or

$$a = a_0 e^{\frac{\alpha}{12}(T_0+t)^4 - \frac{\beta}{2}(T_0+t)^2 + \gamma(T_0+t)} .\tag{18}$$

As  $H$  vanishes at  $t = 0$ ,  $a$  has a minimum there. Then the universe is shrinking when  $t < 0$  and expanding when  $t > 0$ . Since

$$\dot{H} = \alpha(T_0 + t)^2 - \beta ,\tag{19}$$

$\dot{H}$  vanishes at

$$t = t_{\pm} \equiv -T_0 \pm \sqrt{\frac{\beta}{\alpha}} > 0 .\tag{20}$$

Hence,  $w_{\text{eff}}$  (14) is greater than  $-1$  when  $t_- < t < t_+$  (non-phantom phase) and less than  $-1$  when  $0 < t < t_-$  or  $t > t_+$  (phantom phase). There occurs the phantom inflation when  $0 < t < t_-$  and late-time acceleration when  $t > t_+$ . We should also note there does not occur the Big Rip singularity in the solution (18) and  $w_{\text{eff}}$  goes to  $-1$  in the limit of  $t \rightarrow \infty$ . Thus, the model (16) may provide a unification of the inflation generated by phantom and the late time phantom acceleration of the universe.

As a second example, we consider the model given by

$$f(\phi) = h_0 + h_1 \sin(\nu\phi) ,\tag{21}$$

with constants  $h_0$ ,  $h_1$ , and  $\nu$ , which give

$$\begin{aligned}\omega(\phi) &= -\frac{2h_1\nu}{\kappa^2} \cos(\nu\phi) , \\ V(\phi) &= \frac{3}{\kappa^2} (3h_0^2 + 6h_0h_1 \sin(\nu\phi) + h_1\nu \cos(\nu\phi) + h_1^2 \sin^2(\nu\phi)) .\end{aligned}\tag{22}$$

The Hubble rate  $H$  is given by

$$H = h_0 + h_1 \sin(\nu t) ,\tag{23}$$

which is oscillating. When  $h_0 > h_1 > 0$ ,  $H$  is always positive and the universe is expanding. Since

$$\dot{H} = h_1\nu \cos(\nu t) ,\tag{24}$$

when  $h_1\nu > 0$ ,  $w_{\text{eff}}$  (14) is greater than  $-1$  (non-phantom phase) when

$$\left(2n - \frac{1}{2}\right)\pi < \nu t < \left(2n + \frac{1}{2}\right)\pi ,\tag{25}$$

and less than  $-1$  (phantom phase) when

$$\left(2n + \frac{1}{2}\right) \pi < \nu t < \left(2n + \frac{3}{2}\right) \pi . \quad (26)$$

In (25) and (26),  $n$  is an integer. Hence, in the model (22), there occur multiply oscillations between phantom and non-phantom phases. It could be that our universe currently corresponds to late-time acceleration phase in such oscillatory regime.

The third example is given by scalar function :

$$f(\phi) = h_0 \left( \frac{1}{\phi} + \frac{1}{t_s - \phi} \right) , \quad (27)$$

with constants  $h_0$  and  $t_s$ , which give

$$\begin{aligned} \omega(\phi) &= -\frac{2h_0 t_s (2\phi - t_s)}{\kappa^2 \phi^2 (t_s - \phi)^2} , \\ V(\phi) &= \frac{h_0 t_s \{2\phi + (3h_0 - 1)t_s\}}{\kappa^2 \phi^2 (t_s - \phi)^2} . \end{aligned} \quad (28)$$

Then the Hubble rate and the scale factor  $a$  are given by,

$$H = h_0 \left( \frac{1}{t} + \frac{1}{t_s - t} \right) , \quad a = a_0 \left( \frac{t}{t_s - t} \right)^{h_0} , \quad (29)$$

This was obtained from the two scalars model in [4]. As  $a = 0$  at  $t = 0$ , the universe starts at  $t = 0$ . Note that there is a Big Rip type singularity at  $t = t_s$ . Since

$$\dot{H} = \frac{h_0 t_s (2t - t_s)}{t^2 (t_s - t)^2} , \quad (30)$$

when  $0 < t < t_s/2$ , the universe is in non-phantom phase but when  $t_s/2 < t < t_s$ , it is in phantom phase. In fact  $w_{\text{eff}}$  (14) is greater than  $-1$  when  $0 < t < t_s/2$  and less than  $-1$  when  $t_s/2 < t < t_s$ . Hence, again the unified phantom inflation/acceleration universe may emerge.

As a fourth example, we now consider

$$f(\phi) = h_0^2 \left( \frac{1}{t_0^2 - \phi^2} + \frac{1}{\phi^2 + t_1^2} \right) . \quad (31)$$

Here  $h_0$ ,  $t_0$ , and  $t_1$  are positive constants. It is assumed  $t_0 > t_1$ . Then  $\omega(\phi)$  and  $V(\phi)$  are

$$\begin{aligned} \omega(\phi) &= -\frac{8h_0^2 (t_0^2 + t_1^2) \phi \left( \phi^2 - \frac{t_0^2 - t_1^2}{2} \right)}{\kappa^2 (t_0^2 - \phi^2)^2 (\phi^2 - t_1^2)^2} , \\ V(\phi) &= \frac{h_0^2 (t_0^2 + t_1^2)}{\kappa^2 (t_0^2 - \phi^2)^2 (\phi^2 - t_1^2)^2} \left\{ 3h_0^2 (t_0^2 + t_1^2) + 4\phi \left( \phi^2 - \frac{t_0^2 - t_1^2}{2} \right) \right\} . \end{aligned} \quad (32)$$

The Hubble rate  $H$  and the scale factor  $a(t)$  follow as

$$\begin{aligned} H &= h_0^2 \left( \frac{1}{t_0^2 - t^2} + \frac{1}{t^2 + t_1^2} \right) , \\ a &= a_0 \left( \frac{t + t_0}{t_0 - t} \right)^{\frac{h_0^2}{2t_0}} e^{-\frac{h_0^2}{t_1} \text{Arctan} \frac{t_1}{t}} . \end{aligned} \quad (33)$$

Since  $a = 0$  at  $t = -t_0$ , one may regard  $t = -t_0$  corresponds to the creation of the universe. Since

$$\dot{H} = \frac{4h_0^2 (t_0^2 + t_1^2) t \left( t^2 - \frac{t_0^2 - t_1^2}{2} \right)}{(t_0^2 - t^2)^2 (t^2 - t_1^2)^2} , \quad (34)$$

we find  $H$  has two minimum at  $t = t_{\pm} \equiv \pm \sqrt{\frac{t_0^2 - t_1^2}{2}}$  and at  $t = 0$ ,  $H$  has a local maximum. Hence, phantom phase occurs when  $t_- < t < 0$  and  $t > t_+$  and non-phantom phase when  $t_0 > t > t_-$  and  $0 < t < t_+$ . We also note that there is a Big Rip type singularity at  $t = t_0$ .

As is discussed in [11], the solution (7) is stable in the phantom phase but unstable in the non-phantom phase. The instability becomes very large when crossing  $w = -1$ . In order to avoid this problem, one may consider two scalar model. In case of one scalar model, the large instability occurs since the coefficient of the kinetic term  $\omega(\phi)$  in (1) vanishes at the crossing  $w = -1$  point. In the two scalar model, we can choose the corresponding coefficients do not vanish anywhere. Then we may expect that such a divergence of the instability would not occur, which we now check explicitly.

We now consider two scalar model like

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \omega(\phi) \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \eta(\chi) \partial_\mu \chi \partial^\mu \chi - V(\phi, \chi) \right\} . \quad (35)$$

Here  $\eta(\chi)$  is a function of the scalar field  $\chi$ . The FRW equations give

$$\omega(\phi) \dot{\phi}^2 + \eta(\chi) \dot{\chi}^2 = -\frac{2}{\kappa^2} \dot{H} , \quad V(\phi, \chi) = \frac{1}{\kappa^2} (3H^2 + \dot{H}) . \quad (36)$$

Then if

$$\omega(t) + \eta(t) = -\frac{2}{\kappa^2} f'(t) , \quad V(t, t) = \frac{1}{\kappa^2} (3f(t)^2 + f'(t)) , \quad (37)$$

the explicit solution follows

$$\phi = \chi = t , \quad H = f(t) . \quad (38)$$

One may choose that  $\omega$  should be always positive and  $\eta$  be always negative, for example

$$\begin{aligned} \omega(\phi) &= -\frac{2}{\kappa^2} \left\{ f'(\phi) - \sqrt{\alpha^2 + f'(\phi)^2} \right\} > 0 , \\ \eta(\chi) &= -\frac{2}{\kappa^2} \sqrt{\alpha^2 + f'(\chi)^2} < 0 . \end{aligned} \quad (39)$$

Here  $\alpha$  is a constant. Define a new function  $\tilde{f}(\phi, \chi)$  by

$$\tilde{f}(\phi, \chi) \equiv -\frac{\kappa^2}{2} \left( \int d\phi \omega(\phi) + \int d\chi \eta(\chi) \right) , \quad (40)$$

which gives

$$\tilde{f}(t, t) = f(t) . \quad (41)$$

If  $V(\phi, \chi)$  is given by using  $\tilde{f}(\phi, \chi)$  as

$$V(\phi, \chi) = \frac{1}{\kappa^2} \left( 3\tilde{f}(\phi, \chi)^2 + \frac{\partial \tilde{f}(\phi, \chi)}{\partial \phi} + \frac{\partial \tilde{f}(\phi, \chi)}{\partial \chi} \right) , \quad (42)$$

the FRW and the scalar field equations are also satisfied:

$$\begin{aligned} 0 &= \omega(\phi)\ddot{\phi} + \frac{1}{2}\omega'(\phi)\dot{\phi}^2 + 3H\omega(\phi)\dot{\phi} + \frac{\partial\tilde{V}(\phi,\chi)}{\partial\phi}, \\ 0 &= \eta(\chi)\ddot{\chi} + \frac{1}{2}\eta'(\chi)\dot{\chi}^2 + 3H\eta(\chi)\dot{\chi} + \frac{\partial\tilde{V}(\phi,\chi)}{\partial\chi}. \end{aligned} \quad (43)$$

In case of one scalar model, the instability becomes infinite at the crossing  $w = -1$  point (from higher than phantom value), since the coefficient of the kinetic term  $\omega(\phi)$  in (1) vanishes at the point. In the two scalar model in (35), the coefficients  $\omega(\phi)$  and  $\eta(\phi)$  do not vanish anywhere, as in (39). Then we may expect that such a divergence of the instability does not occur.

By introducing the new quantities,  $X_\phi$ ,  $X_\chi$ , and  $Y$  as

$$X_\phi \equiv \dot{\phi}, \quad X_\chi \equiv \dot{\chi}, \quad Y \equiv \frac{\tilde{f}(\phi,\chi)}{H}, \quad (44)$$

the FRW equations and the scalar field equations (43) are:

$$\begin{aligned} \frac{dX_\phi}{dN} &= -\frac{\omega'(\phi)}{2H\omega(\phi)}(X_\phi^2 - 1) - 3(X_\phi - Y), \\ \frac{dX_\chi}{dN} &= -\frac{\eta'(\chi)}{2H\eta(\chi)}(X_\chi^2 - 1) - 3(X_\chi - Y), \\ \frac{dY}{dN} &= \frac{1}{2\kappa^2 H^2} \{X_\phi(X_\phi Y - 1) + X_\chi(X_\chi Y - 1)\}. \end{aligned} \quad (45)$$

Here  $d/dN \equiv H^{-1}d/dt$ . In the solution (38),  $X_\phi = X_\chi = Y = 1$ . The following perturbation may be considered

$$X_\phi = 1 + \delta X_\phi, \quad X_\chi = 1 + \delta X_\chi, \quad Y = 1 + \delta Y. \quad (46)$$

Hence

$$\frac{d}{dN} \begin{pmatrix} \delta X_\phi \\ \delta X_\chi \\ \delta Y \end{pmatrix} = M \begin{pmatrix} \delta X_\phi \\ \delta X_\chi \\ \delta Y \end{pmatrix}, \quad M \equiv \begin{pmatrix} -\frac{\omega'(\phi)}{H\omega(\phi)} - 3 & 0 & 3 \\ 0 & -\frac{\eta'(\chi)}{H\eta(\chi)} - 3 & 3 \\ \frac{1}{2\kappa^2 H^2} & \frac{1}{2\kappa^2 H^2} & \frac{1}{\kappa^2 H^2} \end{pmatrix}. \quad (47)$$

The eigenvalues of the matrix  $M$  are given by solving the following eigenvalue equation

$$\begin{aligned} 0 &= \left(\lambda + \frac{\omega'(\phi)}{H\omega(\phi)} + 3\right) \left(\lambda + \frac{\eta'(\chi)}{H\eta(\chi)} + 3\right) \left(\lambda - \frac{1}{\kappa^2 H^2}\right) \\ &\quad + \frac{3}{2\kappa^2 H^2} \left(\lambda + \frac{\omega'(\phi)}{H\omega(\phi)} + 3\right) + \frac{3}{2\kappa^2 H^2} \left(\lambda + \frac{\eta'(\chi)}{H\eta(\chi)} + 3\right). \end{aligned} \quad (48)$$

The eigenvalues (48) for the two scalar model are clearly finite. Hence, the instability could be finite. In fact, right on the transition point where  $\dot{H} = f'(t) = 0$  and therefore  $f'(\phi) = f'(\chi) = 0$ , for the choice in (39), we find

$$\omega(\phi) = -\eta(\chi) = \frac{2\alpha}{\kappa^2}, \quad \omega'(\phi) = -\frac{2\ddot{H}}{\kappa^2}, \quad \eta'(\chi) = 0. \quad (49)$$

Then the eigenvalue equation (48) reduces to

$$0 = \lambda^3 + (-A - B + 6)\lambda^2 + (AB - 3A - 3B + 9)\lambda - \frac{3}{2}AB + 9B, \quad A \equiv \frac{\ddot{H}}{\alpha}, \quad B \equiv \frac{1}{\kappa^2 H^2}. \quad (50)$$

Here we have chosen  $\alpha > 0$ . Then the eigenvalues are surely finite, which tells that even if the solution (38) could not be stable, the solution has non-vanishing measure and therefore the transition from non-phantom phase to phantom one can surely occur. We should also note that the solution (38) can be in fact stable. For example, we consider the case  $A, B \rightarrow 0$ . Then Eq.(50) further reduces to

$$0 = \lambda(\lambda + 3)^2 . \quad (51)$$

Then the eigenvalues are given by 0 and  $-3$ . Since there is no positive eigenvalue, the solution (38) is stable in the case.

As an example, we consider  $f(t) = h_0 + h_1 \sin(\nu t)$  in (21). Here it is assumed  $h_0, h_1$ , and  $\nu$  are positive. By choosing  $\alpha = h_1 \nu$  in (39), one finds

$$\begin{aligned} \omega(\phi) &= -\frac{2h_1\nu}{\kappa^2} \left\{ \cos(\nu\phi) - \sqrt{1 + \cos^2(\nu\phi)} \right\} , \quad \eta(\chi) = -\frac{2h_1}{\nu} \kappa^2 \sqrt{1 + \cos^2(\nu\chi)} , \\ \tilde{f}(\phi, \chi) &= h_0 + h_1 \sin(\nu\phi) - \frac{\sqrt{2}}{\nu} \left\{ E\left(1/\sqrt{2}, \nu\phi\right) - E\left(1/\sqrt{2}, \nu\chi\right) \right\} . \end{aligned} \quad (52)$$

Here  $E(k, x)$  is the second kind elliptic integral defined by

$$E(k, x) = \int_0^x dx \sqrt{1 - k^2 \sin^2 x} . \quad (53)$$

Then even in two scalar model, the cosmology is given by (23). Similarly any cosmology (including unified inflation/acceleration) can be realized by using the two scalar model, as in the examples with single scalar field.

Thus, we presented several toy models showing the natural possibility to unify early-time inflation with late-time acceleration via phantom-non-phantom transitions in scalar theory. Much work remains to be done in order to decide if such theoretic possibility is realistic one. In next section, we demonstrate that similar effect is possible also in generalized holographic dark energy.

### III. GENERALIZED HOLOGRAPHIC DARK ENERGY AND UNIFICATION OF PHANTOM INFLATION WITH PHANTOM ACCELERATION.

Let us start from the holographic dark energy model [13] (see also refs.[14] where further support for holographic DE was given). Denote the infrared cutoff by  $L_\Lambda$ , which has a dimension of length. If the holographic dark energy  $\rho_\Lambda$  is given by,

$$\rho_\Lambda = \frac{3c^2}{\kappa^2 L_\Lambda^2} , \quad (54)$$

with a numerical constant  $c$ , the first FRW equation

$$\frac{3}{\kappa^2} H^2 = \rho_\Lambda , \quad (55)$$

can be written as

$$H = \frac{c}{L_\Lambda} . \quad (56)$$

Here it is assumed that  $c$  is positive to assure the expansion of the universe. In (55), we do not include the contribution from the matter. The next question is the choice of infrared cut-off. For instance, identifying it with Hubble parameter does not lead to accelerating universe. Hence, one is forced to consider other choices.

The particle horizon  $L_p$  and future horizon  $L_f$  are defined by

$$L_p \equiv a \int_0^t \frac{dt}{a} , \quad L_f \equiv a \int_t^\infty \frac{dt}{a} . \quad (57)$$



For the FRW metric with the flat spacial part:

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2 . \quad (58)$$

Identifying  $L_\Lambda$  with  $L_p$  or  $L_f$ , one obtains the following equation:

$$\frac{d}{dt} \left( \frac{c}{aH} \right) = \pm \frac{1}{a} . \quad (59)$$

Here, the plus (resp. minus) sign corresponds to the particle (resp. future) horizon. The solution of (59) is given by

$$a = a_0 t^{h_0} , \quad (60)$$

with

$$h_0 = \frac{1}{1 \pm \frac{1}{c}} . \quad (61)$$

Then, in the case  $L_\Lambda = L_f$ , the universe is accelerating ( $h_0 > 1$  or  $w = -1 + 2/3h_0 < -1/3$ ). When  $c > 1$  in the case  $L_\Lambda = L_p$ ,  $h_0$  becomes negative and the universe is shrinking. If the theory is invariant under the change of the direction of time, one may change  $t$  with  $-t$ . Furthermore by properly shifting the origin of time, we obtain, instead of (60),

$$a = a_0 (t_s - t)^{h_0} . \quad (62)$$

This tells us that there will be a Big Rip singularity at  $t = t_s$  (for classification of future, finite-time singularities and list of related references, see [4]). Since the direction of time is changed, the particle horizon becomes a future like one:

$$L_p \rightarrow \tilde{L}_f \equiv a \int_t^{t_s} \frac{dt}{a} = a \int_0^\infty \frac{da}{Ha^2} . \quad (63)$$

By using (14) for (60) and (62), we find

$$w_{\text{eff}} = -1 + \frac{2}{3h_0} . \quad (64)$$

Note that if  $L_\Lambda$  is chosen as a future horizon in (57), the deSitter space

$$a = a_0 e^{\frac{t}{l}} \quad \left( H = \frac{1}{l} \right) \quad (65)$$

can be a solution. Since  $L_f$  is now given by  $L_f = l$ , the holographic dark energy (54) is given by  $\rho_\Lambda = \frac{c^2}{\kappa^2 l^2}$ . When  $c = 1$ , the first FRW equation  $\frac{3}{\kappa^2} H^2 = \rho_\Lambda$  is identically satisfied. If  $c \neq 1$ , the deSitter space is not a solution. If  $L_\Lambda$  is chosen to be the particle horizon, the deSitter solution does not exist, either, since the particle horizon  $L_p$  (57) is not a constant:  $L_p = (1 - e^{\frac{t}{l}})/l$ . Hence, the essentials of holographic dark energy are discussed.

In general,  $L_\Lambda$  could be a combination (a function) of both,  $L_p$ ,  $L_f$  [15]. Furthermore, if the span of life of the universe is finite, the span  $t_s$  can be an infrared cutoff. If the span of life of the universe is finite, the definition of the future horizon  $L_f$  (57) is not well-posed, since  $t$  cannot go to infinity. Then, one may redefine the future horizon as in (63)

$$L_f \rightarrow \tilde{L}_f \equiv a \int_t^{t_s} \frac{dt}{a} = a \int_0^\infty \frac{da}{Ha^2} . \quad (66)$$

Since there can be many choices for the infrared cutoff, in analogy with AdS/CFT one may assume  $L_\Lambda$  is the function of  $L_p$ ,  $\tilde{L}_f$ , and  $t_s$ , as long as these quantities are finite:

$$L_\Lambda = L_\Lambda \left( L_p, \tilde{L}_f, t_s \right) . \quad (67)$$

As an example, we consider the generalized holographic dark energy from above class [15]

$$\frac{L_\Lambda}{c} = \frac{\left( \frac{t_s B(1+h_0, 1-h_0)}{L_p + \tilde{L}_f} \right)^{1/h_0}}{h_0 \left\{ 1 + \left( \frac{t_s B(1+h_0, 1-h_0)}{L_p + \tilde{L}_f} \right)^{1/h_0} \right\}^2} , \quad h_0 > 0 . \quad (68)$$

Here  $B(p, q)$  is a beta-function defined by

$$B(p, q) \equiv \int_0^\infty \frac{dt t^{p-1}}{(1+t)^{p+q}} . \quad (69)$$

Eq.(68) leads to the solution:

$$H = h_0 \left( \frac{1}{t} + \frac{1}{t_s - t} \right) , \quad \text{or} \quad a = a_0 \left( \frac{t}{t_s - t} \right)^{h_0} . \quad (70)$$

In fact, one finds

$$L_p + \tilde{L}_f = a \int_0^{t_s} \frac{dt}{a} = t_s \left( \frac{t}{t_s - t} \right)^{h_0} B(1+h_0, 1-h_0) , \quad (71)$$

and therefore

$$\frac{c}{L_\Lambda} = h_0 \left( \frac{1}{t} + \frac{1}{t_s - t} \right) = H , \quad (72)$$

which satisfies (56). For the solution (70),  $w_{\text{eff}}$  defined in (14) is time-dependent and looks as

$$w_{\text{eff}} = -1 + \frac{2(t_s - 2t)}{3h_0 t_s} . \quad (73)$$

Then  $w_{\text{eff}} = -1$  at  $t = t_s/2$  and we find  $w \rightarrow -1 + 2/(3h_0) > -1$  when  $t \rightarrow 0$  and  $w_{\text{eff}} \rightarrow -1 - 2/(3h_0) < -1$  when  $t \rightarrow t_s$ . Hence, there occurs the crossing of  $w_{\text{eff}} = -1$  in our generalized holographic dark energy model.

One can also include the matter whose parameter of the equation of state is  $w$ :  $\rho_m = w p_m$  into the consideration. In the following, we define  $h_0$  by  $h_0 \equiv (2/3)/(1+w)$ . However, it is assumed that there is an interaction between the holographic matters[16]. The energy-conservation law is taken as follows

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 3H \cdot \frac{4\rho_0}{3h_0} \frac{\left\{ 1 + \left( \frac{t_s B(1+h_0, 1-h_0)}{L_p + \tilde{L}_f} \right)^{1/h_0} \right\}^3}{\left( \frac{t_s B(1+h_0, 1-h_0)}{L_p + \tilde{L}_f} \right)^{2/h_0}} . \quad (74)$$

It is also assumed that

$$\frac{L_\Lambda}{c} = \left( 1 - \frac{\kappa^2 \rho_0}{3h_0^2} \right)^{-1/2} \frac{\left( \frac{t_s B(1+h_0, 1-h_0)}{L_p + \tilde{L}_f} \right)^{1/h_0}}{h_0 \left\{ 1 + \left( \frac{t_s B(1+h_0, 1-h_0)}{L_p + \tilde{L}_f} \right)^{1/h_0} \right\}^2} , \quad (75)$$

The first FRW equation is then modified as  $(H^2/\kappa^2) = \rho_\Lambda + \rho_m$  and we find (70) again and

$$\rho_m = \rho_0 \left( \frac{1}{t} + \frac{1}{t_s - t} \right)^2 . \quad (76)$$

Then the ratio of the energy density  $\rho_m$  of matter with respect to that of the holographic dark energy corresponding to (54) is a constant:

$$\frac{\rho_\Lambda}{\rho_m} = \frac{3h_0^2}{\kappa^2\rho_0} \left( 1 - \frac{\kappa^2\rho_0}{3h_0^2} \right) . \quad (77)$$

Hence, one sees that coincidence problem may be solved in generalized holographic dark energy. Note that similar scenario was proposed in [17], where the ratio of the matter energy density and the holographic energy can be a constant by introducing the interaction, as in (74), between the matter and the holographic energy. For the naive model [17], the effective  $w_{\text{eff}}$  (14) is constant. As shown here, even if  $w_{\text{eff}}$  depends on time, the ratio can be constant. In more general case the ratio between the matter energy density and the dark energy density is not constant [18].

As more general case than (67), we may consider the case that  $L_\Lambda$  depends on the Hubble rate  $H$  and the length scale  $l$  coming from the cosmological constant  $\Lambda = 12/l^2$ , if the cosmological constant does not vanish:

$$L_\Lambda = L_\Lambda \left( L_p, \tilde{L}_f, t_s, H, l \right) \text{ or } L_\Lambda \left( L_p, L_f, t_s, H, l \right) . \quad (78)$$

As an example, the generalized holographic dark energy theory from such class may be considered

$$\frac{c}{L_\Lambda} = \frac{1}{\alpha L_f} \left\{ \alpha + 1 + 2(\alpha^2 - \alpha - 1) \frac{L_f}{\alpha l} + 2(\alpha^3 - 2\alpha^2 + \alpha + 1) \left( \frac{L_f}{\alpha l} \right)^2 \right\} . \quad (79)$$

Here  $\alpha$  is a positive dimensionless parameter. Since

$$\begin{aligned} (\alpha^2 - \alpha - 1)^2 - 2(\alpha^3 - 2\alpha^2 + \alpha + 1) &= -\left(\alpha^2 - \frac{1}{2}\right)^2 - 2\alpha - \frac{3}{4} < 0 \\ \alpha^3 - 2\alpha^2 + \alpha + 1 &= \alpha(\alpha - 1)^2 + 1 > 0 , \end{aligned} \quad (80)$$

$L_\Lambda$  is always positive as long as  $\alpha$  is positive. Then a cosmological solution is given by

$$a = \frac{t^{\alpha+1} e^{\frac{t}{\alpha l}}}{L_0 \alpha \left( 1 + \frac{t}{\alpha l} \right)} , \quad L_f = \frac{t}{\alpha \left( 1 + \frac{t}{\alpha l} \right)} , \quad (81)$$

which leads to

$$H = \frac{1 + \alpha \left( 1 + \frac{t}{\alpha l} \right)^2}{t \left( 1 + \frac{t}{\alpha l} \right)} . \quad (82)$$

As clear from (81), if  $\alpha < 0$ , there occurs the Big Rip like singularity at  $t = -\alpha l$ , where  $a$  diverges. When  $\alpha < 0$ ,  $L_\Lambda$  can be negative, in such a case as  $H = \frac{c}{L_\Lambda}$  if we do not include the matter, the universe is shrinking.

In (81),  $L_0$  is a constant of the integration. When  $t$  is small,  $H$  behaves as the inverse power of  $t$ :

$$H \rightarrow \frac{\alpha + 1}{t} , \quad (83)$$

which shows that the universe is filled with the fluid with  $w = -(\alpha - 1)/(3(\alpha + 1))$ . On the other hand, when  $t$  is large,  $H$  goes to a constant:

$$H \rightarrow \frac{1}{l} , \quad (84)$$

which tells that the universe becomes deSitter space asymptotically. Instead of (79), one may consider the model as

$$\frac{c}{L_\Lambda} = \frac{H}{\alpha + 1} + \frac{1}{L_f} \left\{ 1 + \frac{2(\alpha^2 - \alpha - 1)}{\alpha + 1} \frac{L_f}{\alpha l} + \left( 1 - \frac{\alpha^2(\alpha + 2)}{\alpha + 1} \right) \left( \frac{L_f}{\alpha l} \right)^2 \right\}. \quad (85)$$

Then the solution (81) follows again.

In general, the FRW equation has the following form:

$$\frac{3}{\kappa^2} H^2 = \frac{3c^2}{\kappa^2 L_\Lambda^2} + \rho_m. \quad (86)$$

If one defines the matter energy  $E_m$ , the Casimir energy  $E_c$ , and the Hubble entropy  $S_H$  by

$$E_m \equiv \rho_m L_\Lambda^3, \quad E_c \equiv \frac{3c^2 L_\Lambda}{\kappa^2}, \quad S_H \equiv \frac{18\pi c L_\Lambda^3}{\kappa^2} H, \quad (87)$$

we can rewrite the FRW equation in a Cardy-Verlinde (holographic) form [19]:

$$S_H^2 = (2\pi L_\Lambda)^2 E_c (E_c + E_m). \quad (88)$$

Different from the deSitter case [20], the Hubble entropy  $S_H$  is not a constant and depends on time. In case  $\rho_m = 0$ , by using (56) one has

$$S_H = \frac{18\pi c^2 L_\Lambda^2}{\kappa^2} = \frac{18\pi c^4}{\kappa^2 H^2}. \quad (89)$$

Thus for the model (68), using (70), we find

$$S_H = \frac{18\pi c^4 t^2 (t_s - t)^2}{\kappa^2 h_0^2 t_s^2}, \quad (90)$$

which vanishes at  $t = 0$  and  $t = t_s$ . The maximum of  $S_H$  (90) is obtained when  $t = t_s/2$ . For the generalized models (74) and (75), where matter is included,  $S_H$  (90) is modified by a constant factor:

$$S_H = \left( 1 - \frac{\kappa^2 \rho_0}{3h_0^2} \right)^{-3} \frac{18\pi c^4 t^2 (t_s - t)^2}{\kappa^2 h_0^2 t_s^2}. \quad (91)$$

In case of (79) or (86), it follows

$$S_H = \frac{18\pi c^4 t^2 \left( 1 + \frac{t}{\alpha l} \right)^2}{\kappa^2 \left\{ 1 + \alpha \left( 1 + \frac{t}{\alpha l} \right)^2 \right\}^2}, \quad (92)$$

which vanishes at  $t = 0$  and goes to a constant

$$S_H \rightarrow \frac{18\pi c^4 l^2}{\kappa^2}, \quad (93)$$

when  $t \rightarrow \infty$ . In (92),  $\alpha$  can be negative in general but  $S_H$  is positive. Although the Hubble entropy  $S_H$  is always positive in (89),  $S_H$  (91) can be negative if  $\frac{\kappa^2 \rho_0}{3h_0^2} > 1$ . In such a case, if  $S_H$  gives a upper bound for the entropy of the universe, the entropy should be negative. Such a negative entropy has been observed for phantom era in [21]. Nevertheless, when phantom era is transient in the late-time universe it may occur that the universe entropy remains to be positive [9].

As a little bit complicated example, we may consider

$$\frac{c}{L_\Lambda} = \frac{\alpha}{3}T^3 - \beta T + \gamma. \quad (94)$$

Here  $\alpha$ ,  $\beta$ , and  $\gamma$  are positive constants satisfying

$$\gamma > \frac{2\beta}{3}\sqrt{\frac{\beta}{\alpha}}, \quad (95)$$

and  $T$  is defined by

$$\begin{aligned} T &\equiv \frac{H - \gamma + \frac{4\alpha}{3\beta^2}(H + 3\gamma)\ln\frac{L}{F}}{\frac{\alpha}{3\beta^2}(H + 3\gamma)^2 - \beta - \frac{4\alpha}{3\beta}\ln\frac{L}{F}}, \\ L &\equiv \frac{a(t)}{a(0)}F, \\ F &\equiv \int_{-\infty}^{\infty} e^{-\frac{\alpha}{12}t^4 + \frac{\beta}{2}t^2 - \gamma t} dt. \end{aligned} \quad (96)$$

The condition (96) tells that, as a function of  $T$ ,  $H$  vanishes only once. By defining  $T_0 < 0$  via  $H(T = T_0) = 0$ , one gets

$$\gamma = -\frac{\alpha}{3}T_0^3 + \beta T_0. \quad (97)$$

Then the solution can be given by (17) or (18), again. Thus, the generalized dark energy model (94) may provide a unification of the inflation generated by phantom and the late time phantom acceleration of the universe.

In the model (94), the Hubble entropy  $S_H$  (87) is given by

$$S_H = \frac{18\pi c^4}{\kappa^2 \left\{ \frac{\alpha}{3}(t + T_0)^3 - \beta(t + T_0) + \gamma \right\}^2}, \quad (98)$$

which is always positive. From (97), one finds  $S_H$  diverges at  $t = 0$ . Since  $S_H$  is proportional to  $H^{-2}$ ,  $S_H$  decreases when  $0 < t < t_-$ , increases when  $t_- < t < t_+$ , and decreases again when  $t > t_+$ . In the limit  $t \rightarrow \infty$ ,  $S_H$  vanishes. Therefore, in the phantom phase,  $S_H$  is decreasing and in the non-phantom phase, it is increasing. This finishes the discussion of holographic entropy bounds for generalized holographic dark energy model.

#### IV. DISCUSSION

In summary, it is suggested the scenario where phantom cosmology may be key element at early-time as well as at late-time universe. Specific model under consideration suggests the phantom-non-phantom transitions during the evolution of the universe. This may be easily realized due to presence of scalar coupling in front of kinetic term: this coupling function may change its sign on cosmological scales. As a result, even multiply phantom-non-phantom transitions are possible. The intermediate region between very early and very late universe may correspond to standard (radiation/matter dominated) cosmology.

The generalized holographic dark energy where infrared cutoff is identified with combination of the natural FRW parameters: Hubble rate, particle and future horizons, span of life of the universe (when its life-time is finite) and even with cosmological constant is suggested. The possibility to have the crossing of phantom divide there, as well as solution of coincidence problem (when matter presents) is demonstrated. The holographic entropy bound is obtained and the regime where it may be negative is discussed. It is

interesting that holographic entropy is decreasing in phantom phase in accord with earlier observation of ref.[21]. Finally, the possibility to unify phantom inflation with late-time acceleration is demonstrated also in generalized holographic dark energy. It is clear that in similar way one can suggest unified cosmological scenario for tachyon phantoms and for time-dependent, phantom equations of state.

Much work remains to be done in order to understand if such combined scenario is realistic one: the standard cosmological problems (especially at early universe) should be first discussed. In order to start such investigation one awaits the final answer to fundamental question: Do we currently live in phantom universe? Hopefully, the answer comes soon.

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